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$$\Sigma\alpha^4=(\Sigma\alpha^3)(\Sigma\alpha)-\Sigma\alpha^3\beta=-(\Sigma\alpha^2)(\Sigma\alpha\beta)+\Sigma\alpha^2\beta\gamma=-(\Sigma\alpha^2)(\Sigma\alpha\beta)+(\Sigma\alpha)\alpha\beta\gamma$$
$$=-(\Sigma\alpha^2)(\Sigma\alpha\beta)=2q^2.$$

$$\Sigma\alpha^5=(\Sigma\alpha^4)(\Sigma\alpha)-\Sigma\alpha^4\beta=-(\Sigma\alpha^3)(\Sigma\alpha\beta)+\Sigma\alpha^3\beta\gamma=-(\Sigma\alpha^3)(\Sigma\alpha\beta)+(\Sigma\alpha^2)\alpha\beta\gamma=5rq.$$

$$\therefore 3\Sigma\alpha^2\Sigma\alpha^5=5\Sigma\alpha^3\Sigma\alpha^4.$$

⁂ Solved by G. B. M. Zerr and Elmer Schuyler by computing the values of $\Sigma\alpha^2$, $\Sigma\alpha^3$, $\Sigma\alpha^4$, $\Sigma\alpha^5$ by means of the symmetric functions of the roots.

Solved by L. E. Newcomb by actual determination of the roots α, β, γ, of the given equation.

Also solved by G. W. Greenwood.

GEOMETRY.

226. Proposed by **W. J. GREENSTREET, A. M.**, Editor of The Mathematical Gazette, Stroud, England.

The triangles ABC, $A'B'C'$ are in plane perspective, and the corresponding sides BC, $B'C'$, ..., cut in P, Q, R, respectively. AA', ..., cut the line PQR in $P'Q'R'$, respectively. Show that (PP', QQ', RR') is an involution range.

Solution by **M. E. GRABER, A. M.**, Heidelberg University, Tiffin, Ohio.

$B(PP', QR)=B(P'P, Q'R')$ since they both equal the same range ($B'P$, DC) [D being the intersection of $C'B'$ and AB].

Therefore (PP', QR) and ($P'P$, $Q'R'$) are equicross and (PP', QQ', RR') is an involution range by the theorem that if (AA', BC), ($A'A$, $B'C'$) are equicross the range (AA', BB', CC') will be an involution. [Lachlan's *Modern Pure Geometry*, page 272, Art. 426].

229. Proposed by **F. D. POSEY, A.B.**, San Mateo, Cal., and **G. W. GREENWOOD, M.A.** (Oxon), Lebanon, Ill.

The solutions of problem 219 in the April number, "devise a simple geometric solution of the general quadratic equation," give the roots when they are *real*. Required a construction for the roots when they are *complex*.

Solution by **F. D. POSEY, A. B.**, San Mateo, Cal.

Dr. L. E. Dickson reports a solution on page 93 of the April issue of the MONTHLY which holds when the roots are real.

When the circle on AB does not cut Ox the roots are complex. From the center of the circle AB let fall a perpendicular upon Ox cutting the circle at C and Ox at P. Produce this line to D making $CPD=q+1$. On CD as diameter describe a circle cutting Ox at M and N. Then the roots of the equation are $OP+PM\surd(-1)$ and $OP-PM\surd(-1)$.

Proof. CP computed in terms of p and q is found to be

$$\frac{q+1-\surd[p^2+(q-1)^2]}{2}.$$

$$\therefore PD=CD-CP=\frac{q+1+\sqrt{[p^2+(q-1)^2]}}{2}.\quad \therefore PM^2=CP.PD=-\tfrac{1}{4}p^2+q.$$

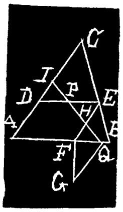
$\therefore (OP+PM\sqrt{-1})(OP-PM\sqrt{-1})=OP^2+PM^2=\frac{1}{4}p^2-\frac{1}{4}p^2+q=q$, and $(OP+PM\sqrt{-1})+(OP-PM\sqrt{-1})=2OP=-p$.

230. Proposed by **SAUL EPSTEEN**, Ph. D.. The University of Chicago.

Cut off a given area S from a given triangle ABC by means of a line passing through a given point P, (i) when P is on a side of ABC, (ii) when P is within ABC ($S<$area ABC). [For the case P outside of ABC see problem 218].

I. Solution by **J. SCHEFFER**, Kee Mar College, Hagerstown, Md.

(i) The point P lies on the side AB. If S is smaller than $\triangle ACP$, find h from the proportion $AP:m=2m:h$, when $S=m^2$.



Lay off h perpendicularly to AB between the sides AC and AB. Connect the point where it strikes AC with P, and $\triangle APQ$ will be the required portion cut off. If $S>ACP$, take BP instead of AP.

(ii) Point P within the triangle.

Draw DE parallel to AB through P. Make parallelogram $LEFA=S$. On F erect perpendicular $FG=DP$, make $GQ=PE$, and connect Q with P extending to I, then will IQ be the required line. For

$$\frac{\triangle FQH}{\triangle PHE}=\frac{PE^2-DP^2}{PE^2}=1-\frac{DP^2}{PE^2}=1-\frac{\triangle DPI}{\triangle PHE}.$$

$\therefore \triangle FAH=\triangle PHE-\triangle DPI$, or $\triangle FQH+\triangle DPI=\triangle PHE$.

$\therefore \triangle FQH+\triangle DPI+AFHPD=\triangle PHE+AFHPD$.

$\therefore \triangle AQI=AFED=S$.

II. Solution by **G. B. M. ZERR**, A. M., Ph. D., Parsons, W. Va.

Let ABC be the given triangle altitude h, P the point on side AB, Q the point within. Lay off $AD=2S/h$ and draw BD; then $ABD=S$.

(i) Draw PD, and parallel to PD draw BM; join PM. The triangle PDM $=$triangle PDB.

$\therefore PDM+ADP=APM=ADP+PDB=ADB=S$.

(ii) Through Q draw HG parallel to AC. Draw HD, and parallel to HD draw BE. Join HE. Then $HED=HBD$, and $AEH=ADB$. Bisect AE at K, and draw KF parallel to AH. Then $AKFH=AEH=ADB$. At K draw KL perpendicular to AC, and make $KL=HQ$, $LM=QF$.

$\therefore KL^2+KM^2=LM^2$. $\therefore HPQ+KNM=QFN$.

$\therefore PAM$ is the required line. For $PAM=AKFH=ADB=S$.

III. Solution by **G. W. GREENWOOD**, M. A. (Oxon), Lebanon, Ill.

The following universal construction is Euclidean, but the proof is based on conics.